On the Asymmetrizing Cost and Density of Graphs

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A set S of vertices in a graph G with nontrivial automorphism group is *asymmetrizing* if the identity mapping is the only automorphism of G that preserves S as a set. If such sets exist, then their minimum cardinality is the *asymmetrizing cost* $\rho(G)$ of G. For finite graphs the *asymmetrizing density* $\delta(G)$ of G is the quotient of the size of S by the order of G. For infinite graphs $\delta(G)$ is defined by a limit process.

The talk discusses bounds on the asymmetrizing cost, classes of graphs with asymmetrizing density zero, and infinite graphs with finite asymmetrizing cost.

It is easy to construct graphs with positive asymmetrizing density, unless they are vertex transitive. Hitherto no infinite vertex transitive graphs with $\delta(G) > 0$ seem to have been known. Here we construct connected, infinite vertex transitive cubic graphs of asymmetrizing density $\delta(G) = n^{-1}2^{-n-1}$ for each $n \geq 1$.

We also construct finite vertex transitive cubic graphs of arbitrarily large asymmetrizing cost. The examples are Split Praeger–Xu graphs, for which we provide another characterization.