A fixed point approach for decaying solutions of difference equations

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A boundary value problem associated to the difference equation with advanced argument

$$\Delta\big(a_n\Phi(\Delta x_n)\big) + b_n\Phi(x_{n+p}) = 0, \quad n \geq 1 \qquad (*)$$

is presented, where $\Phi(u) = |u|^\alpha \text{sgn } u$, $\alpha > 0, p$ is a positive integer and the sequences a, b, are positive. We deal with a particular type of decaying solutions of $(*)$, the so-called intermediate solutions, that is solutions x of $(*)$ such that $x_n > 0, \Delta x_n < 0$ for large n and

$$\lim_n x_n = 0, \quad \lim_n x_n^{[1]} = a_n\Phi(\Delta x_n) = -\infty,$$

where $x^{[1]}$ is called the quasidifference of x. In particular, we prove the existence of these type of solutions for $(*)$ by reducing it to a suitable boundary value problem associated to a difference equation without deviating argument. Our approach is based on a fixed point result for difference equations, which originates from existing ones stated in the continuous case, but take into account some peculiarities of the discrete case.