A numerical algorithm for solving problem for a system of essentially loaded differential equations

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In the present paper we consider the following linear boundary value problem for essentially loaded differential equations with multi-point conditions:

$$\frac{dx}{dt} = A(t)x + \sum_{j=1}^{m} M_j(t)\dot{x}(\theta_j) + \sum_{i=0}^{m+1} K_i(t)x(\theta_i) + f(t), \quad t \in (0,T), \tag{1}$$

$$\sum_{i=0}^{m+1} C_i x(\theta_i) = d, \quad d \in R^n, \quad x \in R^n, \tag{2}$$

where the $(n \times n)$ -matrices $A(t)$, $M_j(t)$ $(j = \overline{1,m})$, $K_i(t)$ $(i = \overline{0,m+1})$, and n-vector-function $f(t)$ are continuous on $[0,T]$, C_i $(i = \overline{0,m+1})$ are constant $(n \times n)$ - matrices, and $0 = \theta_0 < \theta_1 < \theta_2 < \ldots < \theta_{m-1} < \theta_m < \theta_{m+1} = T$; $\|x\| = \max\limits_{i=\overline{1,n}} |x_i|$.

Let $C([0,T], R^n)$ denote the space of continuous functions $x : [0,T] \to R^n$ with the norm $||x||_1 = \max\limits_{t\in[0,T]} ||x(t)||$.

A solution to problem (1), (2) is a continuously differentiable on $(0,T)$ function $x(t) \in C([0,T], R^n)$ satisfying the essentially loaded differential equations (1) and the multi-point condition (2).

We offer algorithm for solving to linear multi-point boundary value problem for essentially loaded differential equations (1), (2). Using the properties of essentially loaded differential equation and assuming the invertibility of the matrix compiled through the coefficients at the values of the derivative of the desired function at load points, we reduce the considered problem to a multi-point boundary value problem for loaded differential equations. The parameterization method [1] is used for solving this problem. The linear boundary value problem for loaded differential equations is reduced to equivalent problem consisting the Cauchy problems for system of ordinary differential equations with parameters in subintervals, multi-point condition and continuity conditions. At fixed values of parameters the Cauchy problem for system of ordinary differential equations in subinterval has a unique solution. This solution is represented with fundamental matrix of system. Using these representations we compile a system of linear algebraic equations with respect to parameters. We proposed algorithm for finding of numerical solution to the equivalent problem [2]. This algorithm includes the numerical solving of the Cauchy problems for system of the ordinary differential equations and solving of the linear system of algebraic equations.

References

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