A boundary value problem for system of differential equations with piecewise-constant argument of generalized type

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In present communication, at the interval $[0,T]$ we consider the two-point boundary value problem for the system of differential equations with piecewise-constant argument of generalized type

$$\frac{dx}{dt} = A(t)x(t) + D(t)x(\gamma(t)) + f(t), \qquad (1)$$

$$Bx(0) + Cx(T) = d, \qquad d \in R^n, \qquad (2)$$

where $x(t) = col(x_1(t), x_2(t), ..., x_n(t))$ is unknown vector function, the $(n\times n)$ matrices $A(t)$, $D(t)$ and n vector function $f(t)$ are continuous on $[0,T]$; $\gamma(t) = \zeta_j$ if $t \in [\theta_j, \theta_{j+1})$, $j = \overline{0, N-1}$; $\theta_j \le \zeta_j \le \theta_{j+1}$ for all $j = 0, 1, ..., N-1$; $0 = \theta_0 < \theta_1 < ... < \theta_{N-1} < \theta_N = T$; B, C are constant matrices, and d is constant vector.

A solution to problem (1), (2) is a vector function $x(t)$ is continuously differentiable on $[0,T]$, it satisfies the system (1) and boundary condition (2).

Differential equations with piecewise-constant argument of generalized type (DEPCAG) are more suitable for modeling and solving various application problems, including areas of neural networks, discontinuous dynamical systems, hybrid systems, etc.[1-2]. To date, the theory of DEPCAG on the entire axis has been developed and their applications have been implemented. However, the question of solvability of boundary value problems for systems of DEPCAG on a finite interval still remains open.

We are investigated the questions of existence and uniqueness of the solution to problem (1), (2). The parametrization method [3] is applied for the solving to problem (1), (2) and based on the construction and solving system of linear algebraic equations in arbitrary vectors of new general solutions [4]. By introducing additional parameters we reduce the original problem (1), (2) to an equivalent boundary value problem for system of differential equations with parameters. The algorithm is offered of findings of approximate solution studying problem also it is proved its convergence. Conditions of unique solvability to problem (1), (2) are established in the terms of initial data. The results can be used in the numerical solving of application problems.

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References

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