On middle Bol loops and the total multiplication groups

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Let $(Q, \cdot)$ be a loop and $a \in Q$. The mappings $x \to ax, x \to xa, x \to a/x$ are denoted by L_a, R_a, D_a, respectively. The multiplication and total multiplication groups $Mlt(Q) = \{L_a, R_a; a \in Q\}$ and $TMlt(Q) = \{L_a, R_a, D_a; a \in Q\}$ of $(Q, \cdot)$ along with their subgroups $Inn(Q)$ and $TInn(Q)$ - the stabilizers of the unit in $Mlt(Q)$ and $TMlt(Q)$, respectively, are important tools in studying such properties of loops as normality of subloops, solvability, nilpotency etc. It is known that $Mlt(Q)$ is invariant under the isotopy of loops (A. Albert) while $TMlt(Q)$ is invariant under the isostrophy of loops (announced by the author and A. Drapal). Characterizations of the mentioned groups for some classes of loops with inverse properties are obtained, including their representation, general properties and systems of generators for $TInn(Q)$. Necessary and sufficient conditions when $TMlt(Q)$ is nilpotent are given.

An open problem regarding middle Bol loops is if this class includes the class of loops with universal (i.e. invariant under the isotopy of loops) flexibility. If this conjecture is true, then the loops with invariant flexibility under the isostrophy are Moufang loops. It is shown that commutative loops with invariant flexibility under the isostrophy are Moufang loops.