Convergence estimates for abstract second order differential equations with two small parameters and lipschitzian nonlinearities

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In a real Hilbert space H endowed with the scalar product $(\cdot,\cdot)$ and the norm $|\cdot|$, consider the following Cauchy problem:

$$\begin{cases} \varepsilon u''_{\varepsilon\delta}(t) + \delta\, u'_{\varepsilon\delta}(t) + Au_{\varepsilon\delta}(t) + B\big(u_{\varepsilon\delta}(t)\big) = f(t), \quad t \in (0,T), \\ u_{\varepsilon\delta}(0) = u_0, \quad u'_{\varepsilon\delta}(0) = u_1, \end{cases} \tag{$P_{\varepsilon\delta}$}$$

where $A : V \subset H \to H$, is a linear self-adjoint operator, V is a real Hilbert space endowed with the norm $||\cdot||$, B is nonlinear $A^{1/2}$ lipschitzian opeartor, $u_0, u_1, f : [0,T] \to H$ and ε, δ are two small parameters.

We investigate the behavior of solutions $u_{\varepsilon\delta}$ to the problem $(P_{\varepsilon\delta})$ in two different cases:
(i) $\varepsilon \to 0$ and $\delta \geq \delta_0 > 0$, relative to the solutions to the following unperturbed system:

$$\begin{cases} \delta l'_\delta(t) + Al_\delta(t) + B\big(l_\delta(t)\big) = f(t), \quad t \in (0,T), \\ l_\delta(0) = u_0; \end{cases} \tag{P_δ}$$

(ii) $\varepsilon \to 0$ and $\delta \to 0$, relative to the solutions to the following unperturbed system:

$$Av(t) + B\big(v(t)\big) = f(t), \quad t \in [0,T), \tag{P_0}$$