Chain connected pair of a topological space and its subspace

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In this talk we introduce the notion of chain connected set in a topological space given in [1,2,3]. A set C in a topological space X is chain connected in X if for every two elements x and y and every open covering $\mathcal{U}$ of X in X, there exists a chain in $\mathcal{U}$ that connects x and y. And by a chain in $\mathcal{U}$ that connects x and y we understand a finite sequence of elements of $\mathcal{U}$ such that x belongs to first element, y to the last, and any two consecutive elements from the chain have nonempty intersection. Also we introduced the notions of pair of chain separated sets [1] and weakly chain separated sets [3] in a space. The nonempty sets A and B in a topological space X are chain separated in X, if there exists an open covering $\mathcal{U}$ of X in X such that for every point $a \in A$ and every $b \in B$, there is no chain in $\mathcal{U}$ that connects x and y. The nonempty sets A and B in a topological space X are weakly chain separated in X, if for every point $a \in A$ and every $b \in B$, there exists an open covering $\mathcal{U}$ of X in X, such that there is no chain in $\mathcal{U}$ that connects x and y. Clearly, if two sets are chain separated in a topological space, then they are weakly chain separated in the same space. We give an example of weakly chain separated sets in a topological space which are not chain separated in the same space. Then we study the properties of these sets.

Moreover we give a criteria for chain connected set in a topological space by using the notions of chain separatedness and weakly chain separatedness. A set C is chain connected in a topological space X if and only if it cannot be represented as a union of two chain (weakly chain) separated sets in X. Then we prove the properties of chain connected sets in a topological space by using the notions of chain separatedness and weakly chain separatedness. Furthermore, we give the criteria for two types of topological spaces using the notion of chain. The topological space is totally separated if any two different singletons are weakly chain separated in the space, and it is the discrete if they are chain separated. Moreover, we generalize the notion to a set in a topological space called totally chain separated set. A set C in a topological space X is totally chain separated in X if every pair of singletons of C are weakly chain separated in X. At the end we prove the properties of totally chain separated sets in a topological space. As a consequence, the properties of totally separated spaces are proven using the notion of chain.

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[2] N. Shekutkovski, Z. Misajleski, E. Durmishi, Chain Connectedness, AIP Conference Proceedings, Vol. 2183, 030015-1-030015-4, (2019);

[3] N. Shekutkovski, Z. Misajleski, A. Velkoska, E. Durmishi, Weakly Chain Connected Set In A Topological Space, In a reviewing process.