Self-stabilizing processes

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A self-stabilizing processes $\{Z(t), t \in [t_0, t_1)\}$ is a random process which when localized, that is scaled to a fine limit near a given $t \in [t_0, t_1)$, has the distribution of an $\alpha(Z(t))$-stable process, where $\alpha : \mathbb{R} \to (0, 2)$ is a given continuous function. Thus the stability index near t depends on the value of the process at t. In the case where $\alpha : \mathbb{R} \to (0, 1)$, we first construct deterministic functions which satisfy a kind of autoregressive property involving sums over a plane point set Π. Taking Π to be a Poisson point process then defines a random pure jump process, which we show has the desired localized distributions.

When α may take values greater than 1, convergence of the considered sums may no longer be absolute. We generalize the construction in two stages, firstly by setting up a process based on a fixed point set but taking random signs of the summands, and then randomizing the point set to get a process with the desired local properties.