

Vertex-transitive distance-regular antipodal covers of complete graphs

Ludmila Tsiovkina

IMM UB RAS

l.tsiovkina@gmail.com

Distance-regular antipodal covers of complete graphs are closely related to various important algebraic, geometric, or combinatorial objects, such as generalized Hadamard matrices, projective planes, generalized quadrangles, divisible designs, and codes. A recent surge of interest for their study is motivated by their applications in discrete geometry and quantum information theory, since those covers that are abelian turn out to be a potential source of new sets of equiangular lines (ETFs, SIC-POVMs). The general problem of classification of all distance-regular antipodal covers of complete graphs seems to be unsolvable. Nevertheless, a promising task in this direction is to describe vertex-transitive representatives, since they admit group-theoretic characterisations. To date, the following vertex-transitive distance-regular antipodal covers of complete graphs have been classified: (i) covers with distance-transitive automorphism groups (complete description); (ii) covers with arc-transitive automorphism groups (almost complete description). Much less is known in general case.

The aim of this talk is to present a classification of edge-transitive distance-regular antipodal covers of complete graphs. The automorphism group of such a cover is transitive, and by a combination of results of Kantor and Burnside, it induces either a 2-transitive almost simple group, or an affine 2-homogeneous group on the set of fibres. Using the classification of finite 2-transitive permutation groups, we will prove that every such cover with $\mu > 1$ is either arc-transitive, or a Cayley graph whose automorphism group induces a one-dimensional affine permutation group on the set of its fibres. Then we will present a general construction of covers with antipodality index greater than 2 in the almost simple case in terms of graphs of basis relations of some association schemes related to quasi-simple groups.

We will also discuss some recent results on classification of abelian distance-regular antipodal covers of a complete graph which possess a transitive group of automorphisms that induces an almost simple primitive rank 3 permutation group on the set of fibres of the corresponding cover.

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