Alternating links, rational balls, and tilings

Brendan Owens

University of Glasgow

brendan.owens@glasgow.ac.uk

Josh Greene

Boston College

joshua.greene@bc.edu

If an alternating knot is a slice of a knotted 2-sphere, then it follows from work of Greene and Jabuka, using Donaldson's diagonalisation combined with Heegaard Floer theory, that the flow lattice L of its Tait graph admits an embedding in the integer lattice $\mathbb{Z}^n$ of the same rank which is "cubiquitous": every unit cube in $\mathbb{Z}^n$ contains an element of L. The same is true more generally for an alternating link whose double branched cover bounds a rational homology 4-ball. This results in an upper bound on the determinant of such links. In this talk I will describe recent joint work with Josh Greene in which we classify alternating links for which this upper bound on determinant is realised. This makes use of Minkowski's conjecture, proved by Hajós in 1941, which states that every lattice tiling of $\mathbb{R}^n$ by cubes has a pair of cubes which share a facet.