

Multiple Orthogonal Polynomials and Random Walks

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Given a non-negative Jacobi matrix describing higher order recurrence relations for multiple orthogonal polynomials of type II and corresponding linear forms of type I, a general strategy for constructing a pair of stochastic matrices, dual to each other, is provided. The corresponding Markov chains (or 1D random walks) allow, in one transition, to reach for the N -th previous states, to remain in the state or reach for the immediately next state. The dual Markov chains allow, in one transition, to reach for the N -th next states, to remain in the state or reach for immediately previous state. The connection between both dual Markov chains is discussed at the light of the Poincaré's theorem on ratio asymptotics for homogeneous linear recurrence relations and the Christoffel–Darboux formula within the sequence of multiple orthogonal polynomials and linear forms of type I.

The Karlin–McGregor representation formula is extended to both dual random walks, and applied to the discussion of the corresponding generating functions and first-passage distributions. Recurrent or transient character of the Markov chain is discussed. Steady state and some conjectures on its existence and the relation with mass points are also given.

The Jacobi–Piñeiro multiple orthogonal polynomials are taken as a case study of the described results.