Loewner's theorem for maps on operator domains

Michiya Mori
RIKEN
michiya.mori@riken.jp

The classical Loewner's theorem states that operator monotone functions on real intervals are described by holomorphic functions on the upper half-plane. We characterize local order isomorphisms on operator domains by biholomorphic automorphisms of the generalized upper half-plane, which is the collection of all operators with positive invertible imaginary part. We describe such maps in an explicit manner, and examine properties of local order isomorphisms. Moreover, in the finite-dimensional case, we explain that every order embedding of a matrix domain is a homeomorphic order isomorphism onto another matrix domain. This is joint work with Peter Šemrl (University of Ljubljana).