Local error estimates for the discretization of elliptic problems with Dirac source term

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It is well known that, as the solution of elliptic problems with a Dirac measure right-hand side in dimension ≥ 2 is not H^1, the convergence of the finite element solutions is suboptimal in the L^2-norm. The solution is, however, smooth whenever we are far away from the singular source term and we can therefore hope for optimal convergence rate in subregions which are disjoint from a neighbourhood of the singularity. In this work we consider problems where the right hand side is, in dimension 3, the Dirac measure along a curve and, in dimension 2, the punctual Dirac measure. We show a quasi optimal convergence in the H^s-norm, for $s \geq 1$ on subdomains which exclude a neighbourhood of the singularity; in the particular case of Lagrange finite elements, an optimal convergence in the H^1-norm is shown on a family of quasi uniform meshes. Our results are obtained using local Nitsche and Schatz-type error estimates, a weak version of Aubin-Nitsche duality lemma and a discrete inf-sup condition. This is a joint work with A. Decoene, Loïc Lacouture and Sébastien Martin.