Optimal control of path-dependent McKean-Vlasov SDEs in infinite dimension

Fausto Gozzi
Luiss University
fgozzi@luiss.it
Andrea Cosso
Università di Bologna (Italy)
andrea.cosso@unibo.it
Idris Kharroubi
Université Paris 1 (France)
idris.kharroubi@upmc.fr
Huyen Pham
Université Paris 7 (France)
pham@math.univ-paris-diderot.fr
Mauro Rosestolato
Università di Lecce (Italy)
mauro.rosestolato@gmail.com

We study the optimal control of path-dependent McKean-Vlasov equations valued in Hilbert spaces motivated by non Markovian mean-field models driven by stochastic PDEs. We first establish the well-posedness of the state equation, and then we prove the dynamic programming principle (DPP) in such a general framework. The crucial law invariance property of the value function V is rigorously obtained, which means that V can be viewed as a function on the Wasserstein space of probability measures on the set of continuous functions valued in Hilbert space. We then define a notion of pathwise measure derivative, which extends the Wasserstein derivative due to P.L.Lions (2006), and prove a related functional Itô formula in the spirit of Dupire (2009) and Wu-Zhang (2020). The Master Bellman equation is derived from the DPP by means of a suitable notion of viscosity solution. We provide different formulations and simplifications of such a Bellman equation notably in the special case when there is no dependence on the law of the control.