

On some fractional queues

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We show some recent advances in the study of fractional queueing models such as fractional M/M/1 queues and fractional Erlang queues $M/E_k/1$. We also focus on a fractional M/M/1 queue with catastrophes. Starting from fractional M/M/1 queues, we study the transient behaviour, in which the time-change plays a key role. An alternative expression for the transient distribution of the fractional M/M/1 model is provided. The state probabilities for the fractional queue with catastrophes, the distributions of the busy period for fractional queues without and with catastrophes and the distribution of the time of the first occurrence of a catastrophe are also obtained.

Furthermore, we introduce a fractional generalization of the Erlang Queues $M/E_k/1$. Such process is obtained through a time-change via inverse stable subordinator of the classical queue process. The fractional Kolmogorov forward equation for such process is considered, then we use such equation to obtain an interpretation of this process in the queueing theory context. We give some results such as the transient state probabilities and some features of this fractional queue model, the mean queue length, the distribution of the busy periods and some conditional distributions of the waiting times.

Finally, we also show some results of the study of a fractional $M/M/\infty$ queueing system constructed as a suitable time-changed birth–death process.