

Polynomial Approximation of the Number of Possible Final Positions of a Random Walk for a Certain Class of Metric Digraphs

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Let us consider a finite compact metric graph (see [1] and references therein), that is, a one-dimensional CW complex and a random walk on it. The main unlikeness with the common graph is that the final position of a walk can be any point on an edge of a metric graph, and not only one of the vertices. Let one point start its move along the graph from a chosen vertex at the initial moment of time. The passage time for each individual edge is fixed. In each vertex, the point with some non-zero probability selects one of the outgoing edges for further movement. Backward turns on the edges are prohibited in this model. Our aim is to analyze an asymptotics of the number of possible final position of such random walk as time increases. The only assumption about the probabilities of choosing an edge is that it is non-zero for all edges, i.e. a situation of a general position. Such random walk could naturally arise while studying the dynamical systems on various networks. The asymptotics for finite compact metric non-directed graphs was constructed earlier by V.L. Chernyshev, in collaboration with A.I. Shafarevich and A.A. Tolchennikov. A polynomial that asymptotically approximates the number of possible final positions of a random walk on an undirected metric graph with incommensurable edges was described with the help of multiple Barnes-Bernoulli polynomials (see [2] for details).

Recently the asymptotics was constructed for a certain class of strongly connected directed graphs (see [3]). This class has been called one-way Sperner graphs. For them it was proved that the asymptotics of the number of possible final positions of a random walk on a metric graph grows polynomially. The degree of a polynomial is equal to the number of incommensurable oriented cycles minus one. The leading coefficient is determined by the product of the lengths of such cycles and the sum of the lengths of all the edges of the graph. Computer simulations show that this result is valid not only for one-way Sperner graphs.

1. Berkolaiko G., Kuchment P., *Introduction to Quantum Graphs*, Mathematical Surveys and Monographs, V. 186 AMS, 2014.

2. Chernyshev V.L., Tolchennikov A.A., *The Second Term in the Asymptotics for the Number of Points Moving Along a Metric Graph*, Regular and Chaotic Dynamics, 2017, vol. 22, no. 8, pp. 937–948.

3. Chernyshev V.L., Tolchennikov A.A., *Asymptotics of the Number of Endpoints of a Random Walk on a Certain Class of Directed Metric Graphs*. Russian Journal of Mathematical Physics, 2021, 28, 2. *In press*.