MODELING, APPROXIMATION, AND ANALYSIS OF PARTIAL DIFFERENTIAL EQUATIONS INVOLVING SINGULAR SOURCE TERMS (MS - ID 39)

A priori error estimates of regularized elliptic problems

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Approximations of the Dirac delta distribution are commonly used to create sequences of smooth functions approximating nonsmooth (generalized) functions, via convolution. We show a-priori rates of convergence of this approximation process in standard Sobolev norms, with minimal regularity assumptions on the approximation of the Dirac delta distribution. The application of these estimates to the numerical solution of elliptic problems with singularly supported forcing terms allows us to provide sharp H^1 and L^2 error estimates for the corresponding regularized problem. As an application, we show how finite element approximations of a regularized immersed interface method result in the same rates of convergence of its non-regularized counterpart, provided that the support of the Dirac delta approximation is set to a multiple of the mesh size, at a fraction of the implementation complexity. Numerical experiments are provided to support our theories.