d-orthogonal analogs of classical orthogonal polynomials

Emil Horozov

retired

horozov@fmi.uni-sofia.bg

Classical orthogonal polynomial systems of Jacobi, Hermite, Laguerre and Bessel have the property that the polynomials of each system are eigenfunctions of second order ordinary differential operator. According to a classical theorem by Bochner they are the only systems with this property. The othogonality property is equivalent to the 3-term recurrence relation according to the famous Favard-Shohat theorem.

Motivated by Bochner's theorem we are looking for d-orthogonal polynomials (systems that satisfy d+2-term recurrence) that are also eigenfunctions of a differential operator. We call these simultaneous conditions Bochner's property.

Using purely algebraic methods and ideas from the bispectral problem, based on automorphisms of non-commutative algebras we construct polynomial systems with Bochner property.

Many properties of the constructed polynomial systems are obtained quite directly from their construction. In particular they have a number of similarities with the classical orthogonal polynomials, which makes them their natural analog - they have hypergeometric representations, ladder operators, generating functions, they can be presented via Rodrigues formulas, there are Pearson's equations for the weights of their measures, they possess the Hahn's property, i.e. the polynomial system of their derivatives are again orthogonal polynomials, etc. We conjecture that the proposed construction exhausts all systems with Bochner's property. Connections to integrable systems like KP and Toda hierarchies and matrix models will be discussed.