Colourful components in k-caterpillars and planar graphs

Clément Dallard
University of Primorska
clement.dallard@famnit.upr.si
Janka Chlebíková
University of Portsmouth
janka.chlebikova@port.ac.uk

A connected component of a vertex-coloured graph is said to be *colourful* if all its vertices have different colours. By extension, a graph is colourful if all its connected components are colourful. Given a vertex-coloured graph and an integer p, the Colourful Components problem asks whether there exist at most p edges whose removal makes the graph colourful. Bulteau, Dabrowski, Fertin, Johnson, Paulusma, and Vialette (2019) proved that Colourful Components is NP-complete on trees with maximum degree at most 6 and asked whether the same would hold if the maximum degree is at most 5. We show that the problem remains NP-complete on binary 4-caterpillars, ternary 3-caterpillars and quaternary 2-caterpillars, where a *k-caterpillar* is a tree containing a path P such that every vertex is at distance at most k from P. On the other hand, we provide a linear-time algorithm for 1-caterpillars (without restriction on the maximum degree), and thus almost settle the complexity dichotomy on k-caterpillars with respect to the maximum degree. Colourful Components has also been studied in vertex-coloured graphs with a bounded number of colours. Bruckner, Hüffner, Komusiewicz, Niedermeier, Thiel, and Uhlmann (2012) showed that the problem is NP-complete on 3-coloured graphs with maximum degree 6, and Bulteau et al. asked whether the problem would remain NP-complete on graphs with bounded number of colors and maximum degree 3. We answer their question in the affirmative by showing that the problem is NP-complete on 5-coloured planar graphs with maximum degree 4 and on 12-coloured planar graphs with maximum degree 3.