Boundary unique continuation of Dini domains

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Let u be a harmonic function in $\Omega \subset \mathbb{R}^d$. It is known that in the interior, the singular set $\mathcal{S}(u) = \{u = |\nabla u| = 0\}$ is $(d-2)$-dimensional, and moreover $\mathcal{S}(u)$ is $(d-2)$-rectifiable and its Minkowski content is bounded (depending on the frequency of u). We prove the analogue near the boundary for C^1-Dini domains: If the harmonic function u vanishes on an open subset E of the boundary, then near E the singular set $\mathcal{S}(u) \cap \overline{\Omega}$ is $(d-2)$-rectifiable and has bounded Minkowski content. Dini domain is the optimal domain for which ∇u is continuous towards the boundary, and in particular every $C^{1,\alpha}$ domain is Dini. The main difficulty is the lack of monotonicity formula near the boundary of a Dini domain. This is joint work with Carlos Kenig.