Optimization over trace polynomials

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Motivated by recent progress in quantum information theory, this article aims at optimizing trace polynomials, i.e., polynomials in noncommuting variables and traces of their products. A novel Positivstellensatz certifying positivity of trace polynomials subject to trace constraints is presented, and a hierarchy of semidefinite relaxations converging monotonically to the optimum of a trace polynomial subject to tracial constraints is provided. This hierarchy can be seen as a tracial analog of the Pironio, Navascués and Acín scheme [New J. Phys., 2008] for optimization of noncommutative polynomials. The Gelfand-Naimark-Segal (GNS) construction is applied to extract optimizers of the trace optimization problem if flatness and extremality conditions are satisfied. These conditions are sufficient to obtain finite convergence of our hierarchy. The results obtained are applied to violations of polynomial Bell inequalities in quantum information theory. The main techniques used in this paper are inspired by real algebraic geometry, operator theory, and noncommutative algebra.