Critical semilinear fractional elliptic problems involving an inverse fractional operator

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In this talk we will study the existence of positive solutions for a problem related to a higher order fractional differential equation involving a nonlinear term depending on a fractional differential operator,

$$\begin{cases} (-\Delta)^{\alpha} u = \lambda u + (-\Delta)^{\beta} |u|^{p-1} u & \text{in } \Omega, \\ (-\Delta)^{j} u = 0 & \text{on } \partial\Omega, \text{ for } j \in \mathbb{Z},\ 0 \leq j < [\alpha], \end{cases}$$

where Ω is a bounded domain in $\mathbb{R}^N$, $0 < \beta < 1$, $\beta < \alpha < \beta + 1$ and $\lambda > 0$. In particular, we will show study the following fractional elliptic problem,

$$\begin{cases} (-\Delta)^{\alpha-\beta} u = \lambda (-\Delta)^{-\beta} u + |u|^{p-1} u & \text{in} \quad \Omega, \\ u = 0 & \text{on} \quad \partial\Omega, \end{cases}$$

proving existence or nonexistence of positive solutions depending on the parameter $\lambda > 0$, up to the critical value of the exponent p, i.e., for $1 < p \leq 2^*_\mu - 1$ where $\mu := \alpha - \beta$ and $2^*_\mu = \frac{2N}{N-2\mu}$ is the critical exponent of the Sobolev embedding.

The results are mainly collected in the following paper,

Álvarez-Caudevilla, P.; Colorado, E.; Ortega, Alejandro. *Positive solutions for semilinear fractional elliptic problems involving an inverse fractional operator.* Nonlinear Anal. Real World Appl. 2020.