Quadrature error estimates for layer potentials evaluated near curved surfaces in three dimensions

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When numerically solving PDEs reformulated as integral equations, so called layer potentials must be evaluated. The quadrature error associated with a regular quadrature rule for evaluation of a layer potential increases rapidly when the evaluation point approaches the surface and the integral becomes nearly singular. Error estimates are needed to determine when the accuracy is insufficient and a more costly special quadrature method should be utilized.

In this talk, we start by considering integrals over curves in the plane, using complex analysis involving contour integrals, residue calculus and branch cuts, to derive such error estimates. We first obtain error estimates for layer potentials in $\mathbb{R}^2$, for both complex and real formulations of layer potentials, both for the Gauss-Legendre and the trapezoidal rule. By complexifying the parameter plane, the theory can be used to derive estimates also for curves in in $\mathbb{R}^3$. These results are then used in the derivation of the estimates for integrals over surfaces. The estimates that we obtain have no unknown coefficients and can be efficiently evaluated given the discretization of the surface, invoking a local one-dimensional root-finding procedure.

Numerical examples are given to illustrate the performance of the quadrature error estimates. The estimates for integration over curves are in many cases remarkably precise, and the estimates for curved surfaces in $\mathbb{R}^3$ are also sufficiently precise, with sufficiently low computational cost, to be practically useful.