Computing distance-regular graph and association scheme parameters in SageMath with `sage-drg`

Janoš Vidali

University of Ljubljana

`janos.vidali@fmf.uni-lj.si`

The `sage-drg` package [8] for the SageMath computer algebra system has been originally developed for computation of parameters of distance-regular graphs, and its functionality has later been extended to handle general association schemes. The package has been used to obtain nonexistence results for both distance-regular graphs and Q-polynomial association schemes, mostly using the triple intersection numbers technique, see for example [3, 4, 7].

Recently, checks for two new feasibility conditions have been implemented. The first technique, developed by Kodalen and Martin [5], relies on Schönberg's theorem on positive semidefinite functions in S^{m-1} and its application on the minimal idempotents of an association scheme. The implementation of the relevant checks in `sage-drg` allows us to replicate their nonexistence results for several feasible parameter sets for Q-polynomial association schemes.

The second technique derives from Terwilliger's work on P- and Q-polynomial association schemes [6] and has most recently been used by Gavrilyuk and Koolen [1, 2] to obtain some nonexistence and uniqueness results for Q-polynomial distance-regular graphs. The implementation of the relevant procedures in `sage-drg` allows us to generalize their approach and derive nonexistence for many feasible parameter sets of classical distance-regular graphs.

References

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