A numerical solution to a nonlinear boundary value problem for the Fredholm integro-differential equation

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It is considered the boundary value problem (BVP) for the Fredholm IDE

$$\frac{dx}{dt} = f(t,x) + \sum_{k=1}^{m} \varphi_k(t) \int_0^T \psi_k(\tau)x(\tau)d\tau, \quad t \in (0,T), \quad x \in \mathbb{R}^n, \qquad (1)$$

$$g\left[x(0), x(T)\right] = 0, \qquad (2)$$

where $f : [0,T] \times \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ are continuous; the $n \times n$ matrices $\varphi_k(t)$, $\psi_k(\tau)$, $k = \overline{1,m}$, are continuous on $[0,T]$, $\|x\| = \max_{i=\overline{1,n}} |x_i|$.

Denote by $\mathbb{C}\big([0,T], \mathbb{R}^n\big)$ the space of continuous functions $x : [0,T] \to \mathbb{R}^n$ with the norm $\|x\|_1 = \max_{t\in[0,T]} \|x(t)\|$. By a solution to problem (1), (2) we mean a continuously differentiable on $(0,T)$ function $x(t) \in \mathbb{C}\big([0,T], \mathbb{R}^n\big)$ that satisfies equation (1) and boundary condition (2).

Employing regular partition Δ_N (see [4, 5]) of the interval [0,T] the Δ_N general solution $x(\Delta_N, t, \lambda)$ to the linear nonhomogenous Fredholm IDE was introduced in cite1. In [7] the new concept of a general solution to the Fredholm IDE (1) was extended. By substituting the corresponding expressions of $x(\Delta_N, t, \lambda)$ into the boundary condition and continuity conditions of a solution to equation (1) at the interior points of Δ_N we construct a system of nonlinear algebraic equations in parameters. It is proved that the solvability of the BVP is equivalent to the solvability of this system.

In present communication, an algorithm for finding a numerical solution to BVP (1), (2) is proposed. To this end, we use the Dzhumabaev parameterization method [3] and results of [6, 4, 5, 1, 2, 7]. At applying the parameterization method to BVP, the special Cauchy problem for a system of nonlinear Fredholm IDEs with parameters and a system of nonlinear algebraic equations in parameters are the intermediate problems. In this case, iterative methods are used both for solving the special Cauchy problem and for solving the systems of nonlinear algebraic equations. The algorithm for solving the special Cauchy problem includes two auxiliary problems: the Cauchy problems for ordinary differential equations and the evaluation of definite integrals. The accuracy of the method that we propose to solve the BVP (1), (2) depends on the accuracy of methods applied to the auxiliary problems and does not depend on a number of the partition subintervals. To solve the Cauchy problems we use the fourth order Runge-Kutta method and to evaluate definite integrals we use Simpson's formula. Therefor, the accuracy of the numerical solution is definite trough the accuracy of these problems.

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References

[1] D.S. Dzhumabaev, *Convergence of iterative methods for unbounded operator equations* // Mat. Zametki, **41**: 5 (1987), 356–361.

[2] D.S. Dzhumabaev, *On the Conergence of a Modification of the Newton-Kantorovich Method for Closed Operator Equations* // Amer. Math. Soc. Transl., **2** (1989), 95–99.

[3] D.S. Dzhumabaev, *Criteria for the unique solvability of a linear boundary-value problem for an ordinary differential equation* // Comput. Math. Math. Phys., **29** (1989), 34–46.

[4] D.S. Dzhumabaev, *Necessary and Sufficient Conditions for the Solvability of Linear Boundary-Value Problems for the Fredholm Integro-differential Equations* // Ukrainian Math. J., **66**: 8 (2015), 1200–1219.

[5] D.S. Dzhumabaev, *On one approach to solve the linear boundary value problems for Fredholm integro-differential equations* // J. Comput. Appl. Math., **294** (2016), 342–357.

[6] D.S. Dzhumabaev, *New general solutions to linear Fredholm integro-differential equations and their applications on solving the boundary value problems* // J. Comput. Appl. Math., **327** (2018), 79–108.

[7] D.S. Dzhumabaev, S.T. Mynbayeva, *New general solution to a nonlinear Fredholm integro-differential equation* // Eurasian Mathematical Journal., **10:** 4 (2019), 24–33.