

Counterexamples for optimal scaling of Metropolis-Hastings chains with rough target densities

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For sufficiently smooth targets of product form it is known that the variance of a single coordinate of the proposal in RWM (Random walk Metropolis) and MALA (Metropolis adjusted Langevin algorithm) should optimally scale as n^{-1} and as $n^{-1/3}$ with dimension n , and that the acceptance rates should be tuned to 0.234 and 0.574. We establish counterexamples to demonstrate that smoothness assumptions such as $\mathcal{C}^1(\mathbb{R})$ for RWM and $\mathcal{C}^3(\mathbb{R})$ for MALA are indeed required if these guidelines are to hold. The counterexamples identify classes of marginal targets, obtained by perturbing a standard Normal density at the level of the potential (or second derivative of the potential for MALA) by a path of fractional Brownian motion with Hurst exponent H , for which these guidelines are violated. For such targets RWM and MALA proposal variances should optimally be scaled as $n^{-1/H}$ and as $n^{-1/(2+H)}$ and will then obey anomalous optimal acceptance rate guidelines. We will discuss useful heuristic implications of the results. The talk is based on the preprint: <https://arxiv.org/abs/1910.09485>.