Exponential dichotomy conditions for difference equations with perturbed coefficients

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A system of linear difference equations with periodic coefficients is considered

$$y_{n+1} = (A(n) + B(n))y_n, \quad n \in \mathbb{Z}, \tag{1}$$

where $A(n)$ are non-degenerate matrices of size $m \times m$ and the matrix sequence $\{A(n)\}$ is N-periodic, i.e. $A(n+N) = A(n), \quad n \in \mathbb{Z}$. The sequence $\{B(n)\}$ is an N-periodic sequence of perturbations. We assume that the system

$$x_{n+1} = A(n)x_n, \quad n \in \mathbb{Z}, \tag{2}$$

is exponentially dichotomous. As shown in[1], this is equivalent to the fact that there are Hermitian matrices $H(0), H(1), \ldots, H(N-1)$ and a matrix P satisfying the following boundary value problem

$$\begin{cases} H(l) - A^*(l)H(l+1)A(l) = \Big(U_l^*\Big)^{-1} P^* U_l^* U_l P U_l^{-1} \\ -\Big(U_l^*\Big)^{-1}(I-P)^* U_l^* U_l (I-P) U_l^{-1}, \quad l = 0, 1, \ldots, N-1, \\ H(0) = H(N) > 0, \\ H(0) = P^* H(0) P + (I-P)^* H(0)(I-P), \\ P^2 = P, \quad PU_N = U_N P, \end{cases} \tag{3}$$

where U_l is the Cauchy matrix of (2). This criterion is analogous to the criterion of M. G. Krein for the exponential dichotomy of difference equations with constant coefficients [2].

Using the fact that the solution of the boundary value problem (3) is represented as

$$H(l) = \Big(U_l^*\Big)^{-1}\left(\sum_{k=0}^{\infty}\Big(U_N^*\Big)^k P^* \Big(\sum_{i=l}^{N+l-1} U_i^* U_i\Big) P U_N^k\right) U_l^{-1}$$
$$+\Big(U_l^*\Big)^{-1}\left(\sum_{k=1}^{\infty}\Big(U_N^*\Big)^k (I-P)^* \Big(\sum_{i=l}^{N+l-1} U_i^* U_i\Big)(I-P) U_N^k\right) U_l^{-1} = H^-(l) + H^+(l),$$

we can obtain conditions for perturbations $\{B(n)\}$ under which the system (1) is also exponentially dichotomous.

Theorem. *Let $det(A(n)) \neq 0$ and the matrix sequence of perturbations $\{B(n)\}$ satisfy the condition*

$$\max\{\|B(0)\|, \ldots, \|B(N-1)\|\}$$

$$< \left(h^- \left(\sqrt{1 - \frac{1}{h^-}} + 1 \right) \sqrt{h^- \|H(0)\|} + h^+ \left(\sqrt{1 + \frac{1}{h^+}} + 1 \right) \sqrt{h^+ \|H(0)\|} \right)^{-1},$$

where

$$h^- = \max\{\|H^-(0)\|, \|H^-(1)\|, \ldots, \|H^-(N-1)\|\},$$

$$h^+ = \max\{\|H^+(0)\|, \|H^+(1)\|, \ldots, \|H^+(N-1)\|\},$$

then the perturbed system (1) is exponentially dichotomous .

This paper is a continuation of [1, 3–5].

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Literature

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