Khrushchev formulas for orthogonal polynomials

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Since their origin in the early 20th century, the theory of orthogonal polynomials on the unit circle (OPUC) has proved to be intimately connected to harmonic analysis via the so called Schur functions. The beginning of this century has witnessed a renewed interest in this connection due to a revolutionary approach to OPUC by the hand of Sergei Khrushchev, which emphasizes the role of continued fractions and Schur functions. The name "Khrushchev theory", coined by Barry Simon, refers to a body of methods and results on OPUC originated by this new approach, whose cornerstone is the so called "Khrushchev formula". The interest of Schur functions and Khrushchev formulas has been fueled even more by a recently uncovered link between OPUC theory and the study of quantum walks, the quantum version of random walks, where Schur functions are central to develop the quantum version of Pólya's renewal theory. This has led to a very general understanding of Khrushchev formula, susceptible to be applied to a wide range of situations covering not only OPUC, but also orthogonal polynomials on the real line (OPRL), matrix valued measures built out of scalar OPUC and OPRL, as well as matrix valued OPUC and OPRL. This talk will give an overview of these different versions of Khrushchev formula, pointing also to their implications in other areas such as harmonic analysis or quantum renewal theory.