Differential geometry of submanifolds in flag varieties via differential equations

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We give a unified method for the general equivalence problem osculating embeddings

$$\varphi\colon (M, \mathfrak{f}) \to \mathrm{Flag}(V, \phi)$$

from a filtered manifold $(M, \mathfrak{f})$ to a flag variety $\mathrm{Flag}(V, \phi)$. We establish an algorithm to obtain the complete systems of invariants for the osculating maps which satisfy the reasonable regularity condition of constant symbol of type $(\mathfrak{g}_-, \operatorname{gr} V)$. We show the categorical isomorphism between the extrinsic geometries in flag varieties and the (weightedly) involutive systems of linear differential equations of finite type. Therefore we also obtain a complete system of invariants for a general involutive systems of linear differential equations of finite type and of constant symbol.

The invariants of an osculating map (or an involutive system of linear differential equations) are proved to be controlled by the cohomology group $H^1_+(\mathfrak{g}_-, \mathfrak{gl}(V)/\operatorname{Prol}(\mathfrak{g}_-))$, which is defined algebraically from the symbol of the osculating map (resp. involutive system), and which, in many cases (in particular, if the symbol is associated with a simple Lie algebra and its irreducible representation), can be computed by the algebraic harmonic theory, and the vanishing of which gives rigidity theorems in various concrete geometries.