Dynamics of visco-elastic bodies with a cohesive interface

Matteo Negri
University of Pavia
matteo.negri@unipv.it

Riccardo Scala
University of Siena
riccardo.scala@unisi.it

We consider the dynamics of elastic materials with a common cohesive interface (or a domain with a prescribed cohesive fracture). In the bulk, the evolution is provided by linearized elasto-dynamics with Kelvin-Voigt visco-elastic dissipation, while on the interface the evolution is governed by a system of Karush-Kuhn-Tucker depending on the crack opening and on the internal variable. The weak formulation reads

$$\begin{cases} \rho\ddot{u}(t) + \partial_u \mathcal{E}(u(t)) - \langle F(t), u\rangle + \partial_{[u]}\Psi(\xi(t), [u(t)]) + \partial_{\dot{u}}\mathcal{R}(\dot{u}) \ni 0, \\ \dot{\xi}(t)(\xi(t) - |[u(t)]|) = 0 \quad \text{and} \quad |[u(t)]| \leq \xi(t), \\ u(0) = u_0, \ \dot{u}(0) = u_1, \end{cases}$$

where $\mathcal{E}$ is the elastic energy, F is the external force, $\mathcal{R}$ is Kelvin-Voigt visco-elastic dissipation, while Ψ is the interface cohesive potential, concave under loading and quadratic under unloading.

First we provide existence of a time-discrete evolution by means of incremental minimization problems (fully implicit in the displacement) and then its time-continuous limit, which satisfies the energy identity

$$\begin{aligned} \mathcal{E}(u(t)) + \Psi([u(t)], \xi(t)) + \mathcal{K}(\dot{u}(t)) = \mathcal{E}(u_0) + \Psi([u_0], \xi_0) + \mathcal{K}(v_0) + \\ + \int_0^t \langle F(s), \dot{u}(s)\rangle \, ds - \int_0^t \partial_v \mathcal{R}(\dot{u}(s))[\dot{u}(s)] \, ds. \end{aligned}$$

Finally, we discuss the strong formulation of the system, with acceleration in L^2 and equilibrium of forces on the interface.