From matrix pivots to graphs in surfaces: touring combinatorics as guided by partial duals

Iain Moffatt
Royal Holloway, University of London
iain.moffatt@rhul.ac.uk

This talk will be accessible to a general mathematical, non-specialist audience.

The concept of the dual of a graph traces back to the very beginnings of graph theory, and can even be found in work of Euler. Roughy speaking, the dual of a graph drawn on the plane is formed by switching its vertices and faces. (For example, the dual of a cube is an octahedron.) Duals are a foundational and well-known concept in graph theory — most undergraduates will meet them at some point in their studies.

Despite their long history, it has only become apparent that you don't have to form the dual of *all* of the edges of a graph at once, you can just take the dual with respect to *some* of its edges. This results in the idea of a "partial dual" — a concept introduced by S. Chmutov in 2009. It arose from work on the Jones polynomial and knot theory.

With such an advance in understanding of such a fundamental construction, it is perhaps unsurprising that partial duals swiftly led to advances in topological graph theory. However, and perhaps more importantly, it turns out that the partial duals reach far beyond graphs in surfaces, are intimately related to several very different areas of combinatorics. Indeed, the concept of the "partial dual" has appeared in very highly disguised forms in various places in the literature over the last 60 years.

In this talk, I'll survey the various appearances and applications of partial duals in graph theory, pointing out the places where the concept has been hiding in the literature for all these years. Along the way we'll encounter various topics in combinatorics such as pivots of matrices, embedded graphs, circle graphs, the Tutte polynomial, knot theory, pivot minors, chord diagrams, and matroids. The emphasis will be on how the various topics fit together, and on what is to be gained by switching between the various perspectives.