Disoriented homology of surfaces and branched covers of the 4-ball

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An often used construction in low-dimensional topology is to associate to a properly embedded surface $F \subset B^4$ the branched double cover X of B^4 with branch set F. If the surface is obtained by pushing the interior of an embedded surface in S^3 into the interior of B^4, a classical result of Gordon and Litherland states that $H_2(X;\mathbb{Z})$ is isomorphic to $H_1(F;\mathbb{Z})$ and that the intersection pairing of X may be described in terms of a pairing on $H_1(F;\mathbb{Z})$ which is determined by the embedded surface in S^3.

We generalize this result to any surface F by defining a non-standard homology theory $DH_*(F)$ that depends on a description of F in S^3. This homology captures the homological information of X and may be equipped with a pairing on $DH_1(F)$ that corresponds to the intersection pairing of X. This construction also works for closed surfaces.