## On the treewidth of even-hole-free graphs

Ni Luh Dewi Sintiari
*ENS de Lyon*
ni-luh-dewi.sintiari@ens-lyon.fr

Even-hole-free graphs attracted attention lately (see [1]). However, many questions about them remain unanswered: polytime algorithm to color them, or to find a maximum stable set, despite the existence of several decomposition theorems. In general, the class of all even-hole-free graphs has unbounded tree-width, as it contains all complete graphs. Nonetheless, bounding the size of the maximum clique does not turn the class into having bounded treewidth, because there exists a family of even-hole-free graphs with no 4-vertex clique which has unbounded tree-width [2]. We observe that the graph constructed in [2] has unbounded degree *and* contains arbitrarily large clique-minors. We ask whether this is necessary.

We prove that for every graph $G$, if $G$ excludes a fixed graph $H$ as a minor, then $G$ either has small tree-width, or $G$ contains a large wall or the line graph of a large wall as *induced* subgraph. This can be seen as a strengthening of Robertson and Seymour's excluded grid theorem for the case of minor-free graphs. Our theorem implies that every class of even-hole-free graphs excluding a fixed graph as a minor has bounded tree-width. In fact, our theorem applies to a more general class: (theta, prism)-free graphs. Furthermore, we conjecture that even-hole-free graphs of bounded degree have bounded tree-width, and we prove it for subcubic graphs and give a partial result when the maximum degree is four. This conjecture is now proved in [3].

Based on a joint work with Pierre Aboulker, Isolde Adler, Eun Jung Kim, and Nicolas Trotignon.

# References

[1] K. Vušković. Even-hole-free graphs: a survey. Applicable Analysis and Discrete Mathematics, 2010.

[2] N. L. D. Sintiari and N. Trotignon. (Theta, triangle)-free and (even hole, $K_4$)-free graphs. Part 1: Layered wheels. Journal of Graph Theory (early view), 2021.

[3] T. Abrishami, M. Chudnovsky, K. Vušković. Even-hole-free graphs with bounded degree have bounded treewidth. arXiv 2009.01297, 2020.