# Optimal boundary control for steady motions of a self-propelled body in a Navier-Stokes liquid

Ana Leonor Silvestre
*Instituto Superior Técnico / Universidade de Lisboa*
ana.silvestre@math.tecnico.ulisboa.pt

Consider a rigid body $\mathcal{S} \subset \mathbb{R}^3$ immersed in a Navier-Stokes liquid and the motion of the body-fluid interaction system described from a reference frame attached to $\mathcal{S}$. We are interested in steady motions of this coupled system, where the region occupied by the fluid is the exterior domain $\Omega = \mathbb{R}^3 \setminus \mathcal{S}$. An important question that arises in this context is: How can a self-propelled motion of $\mathcal{S}$ with a target velocity $V(x) := \xi + \omega \times x$ be generated in such a way that the drag about $\mathcal{S}$ is minimized?

We solve this problem using boundary controls $v_*$, acting on the whole $\partial\Omega$ or just on a portion $\Gamma$ of $\partial\Omega$. Firstly, an appropriate drag functional is derived from the energy equation of the fluid and the problem is formulated as an optimal control problem.

The drag minimization problem is solved for localized controls, such that supp $v_* \subset \Gamma$, and for tangential controls, i.e, $v_* \cdot n|_{\partial\Omega} = 0$, where $n$ is the outward unit normal to $\partial\Omega$. Under smallness restrictions on the objectives $|\xi|$ and $|\omega|$ and on the boundary controls, we prove the existence of optimal solutions, justify the Gâteaux derivative of the control-to-state map, establish the well-posedness of the corresponding adjoint equations and, finally, derive the first order optimality conditions.

This is joint work with Toshiaki Hishida (Nagoya University, Japan) and Takéo Takahashi (Université de Lorraine, CNRS, Inria, IECL, Nancy, France).