Basic Reproduction Number for Conservation laws

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The famous basic reproduction number R_0 plays a key-role in weighing birth/infection and death/recovery processes in population models. In this work we focus on continuously structured models (e.g. where individuals are characterized by age or size, spatial position, etc. . .) as given by conservation laws.

For constant and time periodic environments, R_0 is defined as the spectral radius of the so-called *next generation operator*, which let's say, maps a distribution of population to the distribution of population of their offspring along the whole life span of the former. Analogously in epidemic models, it maps a distribution of infected population to the distribution of their secondary cases.

On the one hand, for populations with concentrated state at birth, we have developed a systematic limit procedure to get to the basic reproduction number. Specifically, R_0 is computed as the limit of basic reproduction numbers of approximate models of populations with distributed state at birth but tending to concentrate (e.g. from an interval to a singleton). So our approach avoids the often computation of R_0 in a heuristically way. We give several examples obtaining explicit expressions.

On the other hand, for populations with distributed state at birth, the computation of the spectral radius of next generation operators poses, in general, serious obstacles to the effective and efficient determination of R_0. Either we have an explicit formula (e.g. for rank one operators) or we address this problem numerically via suitable reductions of the relevant operators to

matrices, thus computing the sought quantity by solving generalized eigenvalue problems, possibly of large dimension. We also give several examples obtaining good results where in addition we proved the compactness of the corresponding next generation operator.

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