Towards Non-Presentable Models of Homotopy Type Theory

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One important aspect of homotopy type theory is the construction of *models*: $(\infty, 1)$-categories in which we can interpret the axioms of our type theory. Studying various models can help us discern which statements can and cannot be proven with our given axiomatization.

Due to a result by Shulman, we already know that every Grothendieck $(\infty, 1)$-topos is a model for homotopy theory. However, we do not expect all models to be a Grothendieck $(\infty, 1)$-topos and in particular, we anticipate non-presentable models of homotopy type theory.

The goal of this work is to take a first step towards showing the existence of non-presentable models of homotopy type theory, by constructing a non-presentable *elementary* $(\infty, 1)$*-topos* . Elementary $(\infty, 1)$-toposes share many features with Grothendieck $(\infty, 1)$-toposes (such as descent, universes, natural number objects, ...), but are not required to be presentable and thus can include examples that are not Grothendieck $(\infty, 1)$-toposes.

We will construct such examples via the *filter construction*. Generalizing a result from 1-category theory, we prove that for every elementary $(\infty, 1)$-topos $\mathcal{E}$ and filter of subobjects Φ, we can construct an elementary $(\infty, 1)$-topos $\prod_{\Phi} \mathcal{E}$, which is in fact not presentable if the filter Φ is not principal. We will apply this result to the $(\infty, 1)$-category of Kan complexes to construct non-presentable examples of elementary $(\infty, 1)$-toposes.