Exact spectral asymptotics of fractional processes and its applications

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Many results in the theory of Gaussian processes rely on the eigenstructure of the covariance operator. However, eigenproblems are notoriously hard to solve explicitly and closed form solutions are known only in a limited number of cases. In this talk we set up a framework for the spectral analysis of the fractional type covariance operators, corresponding to an important family of processes, which includes the fractional Brownian motion, its noise, the fractional Ornstein–Uhlenbeck process and the integrated fractional Brownian motion. We obtain accurate asymptotic approximations for the eigenvalues and the eigenfunctions. Our results provide a key to several problems, whose solution is long known in the standard Brownian case, but was missing in the more general fractional setting. This includes computation of the exact limits of L^2-small ball probabilities and asymptotic analysis of singularly perturbed integral equations, arising in mathematical physics and applied probability.