Optimal control problem for a repulsive chemotaxis system

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The chemotaxis phenomenon can be understood as the directed movement of living organisms in response to chemical gradients. Keller and Segel [5] proposed a mathematical model that describes the chemotactic aggregation of cellular slime molds. These molds move preferentially towards relatively high concentrations of a chemical substance secreted by the amoebae themselves. Such mechanism is called *chemo-attraction* with production. However, when the regions of high chemical concentration generate a repulsive effect on the organisms, the phenomenon is called *chemo-repulsion*.

In this work, we want to study an optimal control problem for the (repulsive) Keller-Segel model and a bilinear control acting on the chemical equation in a $2D$ and $3D$ domains. The system can be written as:

$$\left\{\begin{array}{rcll} \partial_t u - \Delta u - \nabla\cdot(u\,\nabla v) &=& 0 & \text{in } \Omega\times(0,T),\\ \partial_t v - \Delta v + v &=& u + f\,v\,1_{\Omega_c} & \text{in } \Omega\times(0,T),\\ \partial_{\mathbf{n}} u = \partial_{\mathbf{n}} v &=& 0 & \text{on } \partial\Omega\times(0,T),\\ u(0,\cdot)=u_0\geq 0,\quad v(0,\cdot) &=& v_0 \geq 0 & \text{in } \Omega, \end{array}\right. \tag{1}$$

being $f : Q_c := (0,T)\times\Omega_c \to \mathbb{R}$ (the control) with $\Omega_c \subset \Omega \subset \mathbb{R}^n$ $(n = 2,3)$ the control domain, and the state $u, v : Q := (0,T)\times\Omega_c \to \mathbb{R}_+$ the celular density and chemical concentration, respectively. Here, $\mathbf{n}$ is the outward unit normal vector to $\partial\Omega$.

The existence and uniqueness of global in time weak solution (u, v) for the uncontrolled system is known (see for instance [1, 4]).

In this work we study an optimal control problem subject to a chemo-repulsion system with linear production term, and in which a bilinear control acts injecting or extracting chemical substance on a subdomain of control $\Omega_c \subset \Omega$. Existence of weak solutions are stablished (in the $3D$ case by using a regularity criterion), and, as a consequence, a global optimal solution together with first-order optimality conditions for local optimal solutions are deduced.

The results presented in this talk are based on [2, 3].

References

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