Sets with the Baire Property in Topologies Defined From Vitali Selectors of the Real Line

Venuste Nyagahakwa
University of Rwanda
venustino2005@yahoo.fr

Let $\mathcal{F}$ be the family of all countable dense subgroups of the additive topological group $(\mathbb{R}, +)$ of real numbers, and let V be a Vitali selector related to some element Q of $\mathcal{F}$. According to the generalized version of Vitali's theorem, it is well known that all elements of the family $\mathcal{P} := \{V + q : q \in Q\}$ of translated copies of V by points of Q, do not have the Baire property in $\mathbb{R}$, with respect to the Euclidean topology, and they are not measurable in the Lebesgue sense. In this paper, we consider the topological space $(\mathbb{R}, \tau(V))$, where $\tau(V)$ is a topology having $\mathcal{P}$ as a base. Apart from studying the topological properties of $(\mathbb{R}, \tau(V))$, we also look at the relationship between the families of sets with the Baire property in topologies defined from $\tau(V)$, by using distinct ideals of sets on $\mathbb{R}$. Moreover, we show that for any $Q_i \in \mathcal{F}$, the spaces $(\mathbb{R}, \tau(V_i)), i = 1, 2$, where V_i is Vitali selector related to Q_i, are homeomorphic. We further prove that the families of sets with the Baire property in the spaces $(\mathbb{R}, \tau(V_1))$ and $(\mathbb{R}, \tau(V_2))$ are Baire congruent.

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