Method of energy estimates for studying of singular boundary regimes in quasilinear parabolic equations

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In the cylindrical domain $Q = (0,T) \times \Omega,\ 0 < T < \infty$, where $\Omega \subset R^n$ is a bounded domain such that $\partial\Omega \in C^2$, the following problem is considered:

$$\begin{aligned} &(|u|^{q-1}u)_t - \Delta_p u = 0, \quad p \geqslant q > 0,\\ &u(0,x) = u_0 \ \text{ in } \Omega, \quad u_0 \in L^{q+1}(\Omega),\\ &u(t,x)\Big|_{\partial\Omega} = f(t,x), \end{aligned} \tag{1}$$

where f generates boundary regime with singular peaking, namely,

$$f(t,x) \to \infty \quad \text{as } t \to T, \quad \forall\, x \in\ K \subset \partial\Omega, K \neq \emptyset. \tag{2}$$

Function f is called a localized boundary regime (S-regime) if

$$\overline{\Omega} \setminus \Omega_0 \neq \emptyset, \quad \text{where } \Omega_0 := \left\{ x \in \overline{\Omega} : \sup_{t\to T} u(t,x) = \infty \right\}$$

for an arbitrary weak solution u of problem (1). Sharp conditions of localization of boundary regime were obtained by some version of local energy estimates (see [1] and references therein). Papers [2, 3] are devoted to investigation of the behavior of weak solutions in the case when the blow-up set $\Omega_0 \subset \partial\Omega$ (LS-regime). The precise estimates of the limiting profile of solutions were obtained, namely,

$$\sup_{t\to T} u(t,x) \leqslant \psi(x), \quad x \in \Omega,$$

where function ψ is determined by characteristics of peaking of the boundary regime f.

As an application of these results we study the following parabolic quasilinear equation with a nonlinear absorption term:

$$(|u|^{q-1}u)_t - \Delta_p u = -b(t,x)|u|^{\lambda-1}u, \quad (t,x) \in Q, \quad \lambda > p \geqslant q > 0, \tag{3}$$

Here $b(t,x) \geqslant 0$ is a degenerate absorption potential: $b(t,x) \to 0$ as $t \to T$ $\forall\, x \in \Omega$. Precise upper estimates for all weak solutions of equation (3) near to $t = T$ (limiting profile of solution), depending on the behavior of function b, were obtained in the papers [2, 4]. It is important to underline that the obtained estimates don't depend on initial and boundary values and hold for large solutions of the equation (3) (if they exist).

References

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