Integration the loaded KdV equation in the class of steplike function

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2010 Mathematics Subject Classification: 39A23, 35Q51, 34K13, 34K29.
Keywords : loaded Korteweg-de Vries equation, inverse scattering problem, the class of steplike function, scattering data, Lax pair, eigenvalue, eigenfunction.

It is known, that the Korteweg-de Vries equation can be integrated with Inverse Scattering Method [1]. In the works [2,3], the Korteweg-de Vries equations with a self-consistent source were integrated for a class of initial data of "step" type; in particular, laws of evolution of the scattering data were established. In applications of the method of inverse scattering transformation one looks for pairs of operators B and L such that the equation has some interesting nonlinear evolution equation for functions $u(x,t)$ that occur as potentials in the operator L. For the successful application of the method two further ingredients are needed: 1. the inverse scattering problem must be solved so that the potentials $u(x,t)$ can be reconstructed from scattering data; 2. and that one must be able to determine the evolution of the scattering data with$t$.

In this paper, we will consider the loaded Korteweg-de Vries equation

$$u_t - 6uu_x + u_{xxx} + \gamma(t)u(0,t)u_x = 0, \tag{1}$$

where $u = u(x,t)$, $x \in R$, $t \geq 0$, $\gamma(t)$ - is an arbitrary, continuous function.

The function $u = u(x,t)$ is a sufficiently smooth and tending to its limits steplike ($c > 0$)

$$\int_{-\infty}^{0} (1-x)|u(x,t)|dx + \int_{0}^{\infty} (1+x)\big|u(x,t) - c^2\big|dx + \sum_{k=1}^{3} \int_{-\infty}^{\infty} \left|\frac{\partial^k u(x,t)}{\partial x^k}\right| dx < \infty \tag{2}$$

The equation (1) is considered with initial condition

$$u|_{t=0} = u_0(x), \quad x \in R^1, \tag{3}$$

where $u_0(x)$ function satisfies the conditions ($c > 0$):

1. $\int\limits_{-\infty}^{0} (1-x)|u_0(x)|dx < \infty, \qquad \int\limits_{0}^{\infty} (1+x)|u_0(x) - c^2|dx < \infty,$

2. Suppose that, the equation $-y'' + u_0(x)y = \lambda y, \quad x \in R^1$ has $\lambda_1(0)$, $\lambda_2(0)$, ..., $\lambda_N(0)$ negative eigenvalues.

In this work the solution $u(x,t)$ of the loaded Korteweg-de Vries equation (1) in the class of steplike function (2) with initial condition (3) is obtained via Inverse Scattering Method.

References

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