Limit states of multi-component discrete dynamical systems

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Object. We study models of multicomponent discrete dynamic conflict systems with attractive interaction, which are characterized by a positive value that is called the attractor index. Consider the set of discrete probability measures $\mu_i \in M_1^+(\Omega)$ on finite space $\Omega = \{\omega_1, \dots, \omega_n\}$, $i = \overline{1,m}$. Each of these measures μ_i can be identified with a stochastic vector $\mathbf{p}_i = (p_{ij})_{j=1}^n$, where

$$p_{ij} = \mu_i(\omega_j), \quad i = \overline{1,m}, \quad j = \overline{1,n}.$$

Consider the mapping $\circledast$

$$\{\mathbf{p}_1^t, \mathbf{p}_2^t, \dots, \mathbf{p}_m^t\} \xrightarrow{\circledast, t} \{\mathbf{p}_1^{t+1}, \mathbf{p}_2^{t+1}, \dots, \mathbf{p}_m^{t+1}\}, \tag{1}$$

which generates multi-component discrete dynamical systems with trajectories (1), where the coordinates of each vector $\mathbf{p}_i^t = (p_{ij}^t)_{j=1}^n$ are changed according to equations

$$p_{ij}^{t+1} = \frac{1}{z^t}\big(p_{ij}^t(\theta^t + 1) + \tau_j^t\big), \quad t = 0, 1, \dots. \tag{2}$$

Here $\theta^t = \theta(\mathbf{p}_1^t, \mathbf{p}_2^t, \dots, \mathbf{p}_m^t)$ is a finite positive function, $\mathcal{T}^t = (\tau_j^t)_{j=1}^n$ is a vector with non-negative coordinates (attractor index), and $z^t = \theta^t + 1 + W^t$ is normalizing denominator, $W^t = \sum_{j=1}^n \tau_j^t$.

Main results. ***Theorem 1.*** *Let all coordinates of vector* $\mathbf{w}^t = (w_j^t)_{j=1}^n$, $w_j^t := \frac{\tau_j^t}{W^t}$ *be bounded and monotonic (increase or decrease independently one to other). Then for all* $i = \overline{1,m}$ *there exist*

$$\mathbf{p}_i^\infty = \lim_{t\to\infty} \mathbf{p}_i^t$$

and all limit vectors $\mathbf{p}_i^\infty$ *coincide with the vector* $\mathbf{w}^\infty$, *i.e.*

$$p_{ij}^\infty = \frac{\tau_j^\infty}{W^\infty} \quad \forall j.$$

Let us consider the different variants of attractor index $\mathcal{T}^t$:

$$\tau_j^t := \tau_{j,\min}^t = \min_i \{p_{ij}^t\}, \tag{3}$$

$$\tau_j^t := \tau_{j,\max}^t = \max_i \{p_{ij}^t\}, \tag{4}$$

$$\tau_j^t := \overline{\tau}_j^t = \frac{1}{m}\sum_{i=1}^{n} p_{ij}^t, \tag{5}$$

$$\tau_{j_1}^t = \tau_{j_2}^t > 0, \quad j_1, j_2 = \overline{1,n}. \tag{6}$$

Theorem 2. *Let coordinates of the attractor index $\mathcal{T}^t$ be given by one of the equations (3), (4), (5), (6). Then each trajectory of a dynamic system* (1) *with an initial state* $\{\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_m\}$ *converges to the fixed point* $\{\mathbf{p}_1^\infty, \mathbf{p}_2^\infty, \ldots, \mathbf{p}_m^\infty\}$

$$\mathbf{p}_i^\infty = \lim_{t\to\infty} \mathbf{p}_i^t, \ \forall\ i = \overline{1,m},$$

where the coordinates of $\mathbf{p}_i^\infty$ *have a view:*

$$p_{ij}^\infty = \frac{\tau_j}{W} \ \ \forall\ i = \overline{1,m}, \quad j = \overline{1,n}. \tag{7}$$

Stability. The limit state is unstable in cases (3)-(5), however it is stable only in the case (6), when all limit coordinates are equal to $\frac{1}{n}$.

Application. Such model of dynamic systems can describe the dynamics of real processes. Attractor index can describe a real external influence on a certain system (for example, information influence on a society). System behavior can be controlled or described by setting attractor index which can be exposed to such influence.

REFERENCES

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