Generation of first passage times for diffusion processes: an overview of simulation techniques

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Many biological or physical applications require to simulate random variables with a given probability distribution. The aim of our study is to focus on a particular random variable: the first passage time (FPT) of a diffusion process. We introduce (X_t) the unique solution of the following SDE:

$$dX_t = b(X_t)\,dt + \sigma(X_t)\,dB_t, \quad X_0 = x,$$

where (B_t) stands for a one-dimensional Brownian motion and define τ_L the first passage time through the level L. We propose an overview of several simulation techniques.

- The classical way is to use efficient algorithms for the simulation of sample paths, like discretization schemes. Such methods permit to obtain approximations of the first-passage times as a by-product.
- Another approach based on a random walk on spheroids permit in particular cases to express the first passage time as the limit of a random walk. It suffices therefore to describe precisely the convergence of this stochastic process and to introduce a stopping procedure.
- Finally we present a new rejection sampling algorithm which permits to perform an exact simulation of the first-passage time for general one-dimensional diffusion processes. The main ideas are based both on a previous algorithm pointed out by A. Beskos et G. O. Roberts which uses Girsanov's transformation and on properties of Bessel paths.

References

1. A. Beskos and G.O. Roberts, *Exact simulation of diffusions*, The Annals of Applied Probability, 15(4), 2005.
2. S. Herrmann and C. Zucca, *Exact simulation of the first-passage time of diffusions*, Journal of Scientific Computing, 2019.