Geometry of planar domains and their applications in study of conformal and harmonic mappings

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The famous Riemann Mapping Theorem states that for every simply connected domain $\Omega \neq \mathbb{C}$ containing a point w_0 there exists a essentially unique univalent function f such that $f(0) = w_0$ and $f_z(0) > 0$, that maps the unit disk $\mathbb{D}$ onto Ω.

Open sense-preserving quasiconformal mappings of $\mathbb{D}$ arised as a solutions of linear elliptic partial differential equations of the form

$$\overline{f}_{\bar{z}}(z) = \omega(z) f_z(z), \quad z \in \mathbb{D},$$

where ω is an analytic function from $\mathbb{D}$ into itself, known as a dilatation of f and such that $|\omega(z)| < k < 1$.

A natural generalization of the classical class of normalized univalent functions on $\mathbb{D}$ is the class of sense-preserving univalent harmonic mappings on $\mathbb{D}$ of the form $f = h + \overline{g}$ normalized by $h(0) = g(0) = h'(0) - 1 = 0$.

In the context of univalent, quasiconformal and planar harmonic mappings a problem of convexity, linear convexity, starlikeness, etc. have been intensively studied in the past decades. Additional properties of a planar domains exhibits a very rich geometric and analytic properties.

We discuss behavior of the function f for which some functional are limited to the Both leminiscates, Pascal snail, hyperbola and conchoid of the Sluze. Some appropriate examples are demonstrated.