Seiberg-Witten equations and pseudoholomorphic curves

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Seiberg–Witten equations (SW-equations for short) were proposed in order to produce a new kind of invariant for smooth 4-dimensional manifolds. These equations, opposite to the conformally invariant Yang–Mills equations, are not invariant under scale transformations. So to draw a useful information from these equations one should plug the scale parameter λ into them and take the limit $\lambda \to \infty$.

If we consider such limit in the case of 4-dimensional symplectic manifolds solutions of SW-equations will concentrate in a neighborhood of some pseudoholomorphic curve (more precisely, pseudoholomorphic divisor) while SW-equations reduce to some vortex equations in normal planes of the curve. The vortex equations are in fact static Ginzburg–Landau equations known in the superconductivity theory. So solutions of the limiting adiabatic SW-equations are given by families of vortices in the complex plane parameterized by the point z running along the limiting pseudoholomorphic curve. This parameter plays the role of complex time while the adiabatic SW-equations coincide with a nonlinear $\bar{\partial}$-equation with respect to this parameter.