A glimpse to the Berezin numbers inequality

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A reproducing kernel Hilbert space (RKHS for short) $\mathcal{H} = \mathcal{H}(\Omega)$ is a Hilbert space of complex valued functions on a (nonempty) set Ω, which has the property that point evaluations are continuous i.e. for each $\lambda \in \Omega$ the map $f \mapsto f(\lambda)$ is a continuous linear functional on $\mathcal{H}$. The Riesz representation theorem ensure that for each $\lambda \in \Omega$ there is a unique element $k_\lambda \in \mathcal{H}$ such that $f(\lambda) = \langle f, k_\lambda\rangle$, for all $f \in \mathcal{H}$. The collection $\{k_\lambda : \lambda \in \Omega\}$ is called the reproducing kernel of $\mathcal{H}$. If $\{e_n\}$ is an orthonormal basis for a functional Hilbert space $\mathcal{H}$, then the reproducing kernel of $\mathcal{H}$ is given by $k_\lambda(z) = \sum_n \overline{e_n(\lambda)} e_n(z)$. For $\lambda \in \Omega$, let $\hat{k_\lambda} = \frac{k_\lambda}{\|k_\lambda\|}$ be the normalized reproducing kernel of $\mathcal{H}$. For a bounded linear operator A on $\mathcal{H}$, the function $\widetilde{A}$ defined on Ω by $\widetilde{A}(\lambda) = \langle A\hat{k_\lambda}, \hat{k_\lambda}\rangle$ is the Berezin symbol of A, which firstly have been introduced by Berezin. The Berezin set and the Berezin number of the operator A are defined by

$$\mathbf{Ber}(A) := \{\widetilde{A}(\lambda) : \lambda \in \Omega\} \qquad \text{and} \qquad \mathbf{ber}(A) := \sup\{|\widetilde{A}(\lambda)| : \lambda \in \Omega\},$$

respectively. Namely, the Berezin transform has been investigated in detail for the Toeplitz and Hankel operators on the Hardy and Bergman spaces; it is widely applied in the various questions of analysis and uniquely determines the operator(i.e., for all $\lambda \in \Omega$, $\widetilde{A}(\lambda) = \widetilde{B}(\lambda)$ implies $A = B$).

The objective of this paper is to present a generalized Berezin number inequality and refine the new inequalities. We also present some results of Berezin number inequalities involving f-connection of operators.

References

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